Why a 4:1 TUR is not Enough: The Importance of Analyzing the Probability of False Accept Risk

Introduction

Several organizations and publications reference or insist on maintaining a 4:1 Test Uncertainty Ratio (TUR) without understanding the level of measurement risk. The thought is that if the lab performing the calibrations has standards at least four times better than what they are calibrating, everything is good. ANSI/NCSL Z540.3 – Requirements for Calibration of Measuring and Test Equipment in section 5.3 b) allows for use of a test uncertainty ratio (TUR) equal to or greater than 4:1 when it is not practical to estimate the false accept risk of less than 2%. Then goes on to say objective evidence of nonpracticability of this determination is expected as in an agreement with the customer TUR use.

The 4:1 TUR seems to be a fallback position that many industries have adopted, maybe because they did not understand or want to deal with guard bands. The assumption is that the higher the TUR, the higher the probability that the measuring equipment will have a Probability of False Accept (PFA) of less than 2 %, assuming the measured reading is closer to the nominal value. However, laboratories who insist on a 4:1 TUR might also want to consider the location of the measurement to comply with ISO/IEC 17025:2017 as the standard focuses more on a risk-based approach that takes measurement uncertainty into account.

Many fail to realize what is described in "Introduction to Statistics in Metrology", that using a TUR assumes that all measurement biases have been removed from the measurement process and the measurements involved follow a normal distribution. If there are significant biases that cannot be removed, the TUR will not account for the increased risk.

Section 7.8.6.1 states, "When a statement of conformity to a specification or standard for test or calibration is provided, the laboratory shall document the decision rule employed, taking into account the level of risk (such as false accept and false reject and statistical assumptions) associated with the decision rule employed and apply the decision rule".

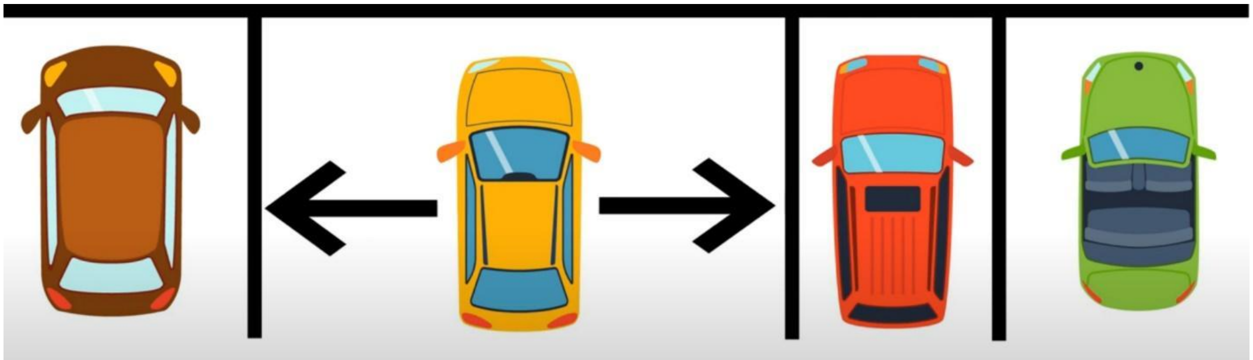

Figure1: Parking Lot Example with Small Calibration Process Uncertainty

We can think about the measurement risk this way. We have a car, and we need to park it between two lines. The lines represent the upper and lower specification limit of our device. The width of our car is the uncertainty, and parking lines are our tolerance specification limits. The probability of us getting a ding or denting another vehicle is our PFA, depending on how centered we are within the parking lines. If we try to park too close to one side, we may risk not being able to open the door, or if we misjudge entirely we may run right into the car in the other lane and cause substantial damage. If we park centered on the line, 50 % of our car will be in the next lane no matter what size our car is.

TUR works the same way. No matter what TUR ratio we have, if we are right at the non-guard band specification limit, there will be at least a 50 % chance that our measurement falls outside of the specified tolerance (see figure 2). This paper is going to discuss TUR, why the location of the measurement matters, PFA, and some common guard banding methods used to assure measurements are compliant and stay within the lines. The examples cited are assumed to be based on discrete measurement at the bench level (Specific Risk).

Nominal Value	10000
Lower specification Limit	9995
Upper Specification Limit	10005
Measured Value	10005
Measurement Error	5
Std. Uncert. (k=1)	0.08
Total Risk	50.00%
Upper Limit Risk	50.00%
Lower Limit Risk	0.00%
TUR =	**31.2296736**
Guard Band LSL	9995.164407
Guard Band USL	10004.83559

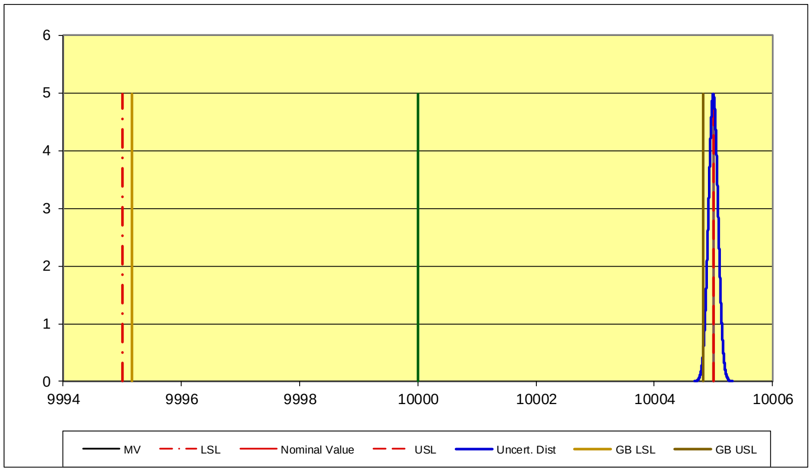

Figure 1: 31.23:1 TUR with a 50 % PFA at the Upper Specification Limit

TUR

TUR or Test Uncertainty Ratio is defined in Section 3.11 of ANSI/NCSL Z540.3 as, "The ratio of the span of the tolerance of a measurement quantity subject to calibration, to twice the 95% expanded uncertainty of the measurement process used for calibration." What the TUR tells us is how much space between the lines we must be "in-tolerance". NCSLI RP-12 states in section 12.3 "The uncertainty in the value or bias, always increases with time since calibration". The recommendation is to analyze the data and set the tolerances so that the calibration supplier has uncertainties low enough to make the statement of conformity, and that any drift be accounted for between calibration cycles.

NCSLI RP-12 Section 12 suggest when developing equipment tolerance, it may be prudent to include uncertainties due to factors that are not normally included in the list of measurement process errors applicable to calibrations. Several organizations are concerned with setting the system to a certain accuracy and making sure the calibration laboratory adjusts within the specification. However, when adjustments occur frequently, it becomes apparent that the tolerances are not appropriate for the device or the cycle time between calibrations is not set appropriately. Setting higher tolerance or specification limits will improve the TUR ratio.

Using a calibration provider with low uncertainties will help raise the TUR ratio. The higher TUR will result in wider acceptance (compliance) limits. Wider acceptance limits give more room to account for the bias increase that will occur between calibrations. However, if the uncertainties are not properly accounted for, the probability of the PFA being higher than 2 % will increase. It is important to consider all sources of measurement uncertainty when determining the time between calibration as well as tolerance limits, and in some cases, the manufacturer's tolerance may not be achievable.

PFA Risk

Nominal Value	10000
Lower specification Limit	9995
Upper Specification Limit	10005
Measured Value	10004.83559
Measurement Error	4.83559
Std. Uncert. (k=1)	0.08
Total Risk	2.00%
Upper Limit Risk	2.00%
Lower Limit Risk	0.00%
TUR =	**31.2296736**
Guard Band LSL	9995.164407
Guard Band USL	10004.83559

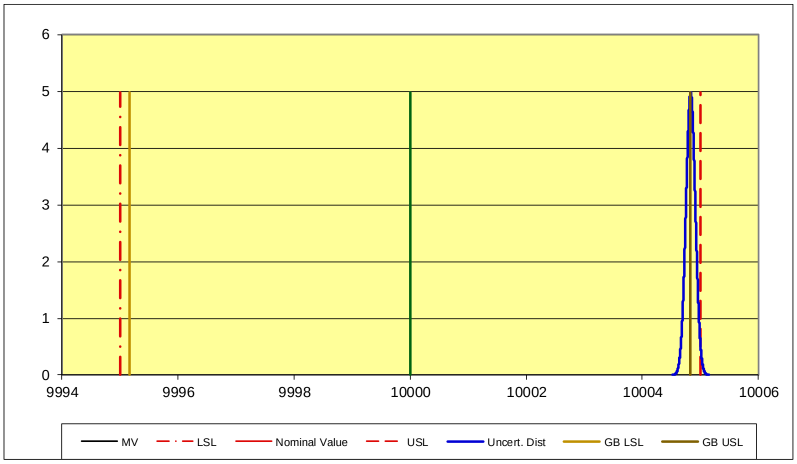

Figure 2: Guard band USL showing a 2 % PFA when Measured Value is at the GB USL

All measurements have a percentage of likelihood of calling something good when it is bad, and something bad when it is good. You might be familiar with the terms consumer's risk and producer's risk. Consumer's risk refers to the possibility of a problem occurring in a consumer-oriented product; occasionally, a product not meeting quality standards passes undetected through a manufacturer's quality control system and enters the consumer market. The Probability of False Accept (PFA) is similar to consumer's risk. It is the likelihood of calling a measurement "good" or stating something is "In Tolerance" when there is a percentage that the measurement is "bad" or "Out of Tolerance".

ANSI/NCSLI sub-clause 5.3 is the tolerance-type test requirement that "the probability that incorrect acceptance decisions (false accept) will result from calibration tests shall not exceed 2 %." With the preponderance of calibrations being of this type, the resources and conditions described by the calibration procedure will require careful evaluation and determination to achieve the measurement uncertainty needed for the calibration process to achieve this allowable probability of false accept." The measurement uncertainty must be accounted for, and the acceptance limits must be calculated to ensure the likelihood of the measurement being "Out of Tolerance" does not exceed 2 %.

The entire purpose of analyzing the PFA is to ensure your measurements are "In Tolerance" with risk that does not exceed 2 %. And why just knowing you have a 4:1 TUR without analyzing the PFA regarding the location of the measurement is not enough to minimize measurement risk as shown in Figure 3. Figure 3 shows the upper and lower guard banded limits to ensure a PFA of 2 % or less. If the measured value is not within the guard band limits, the PFA will be higher than 2 %.

Location of the Measurement

Nominal Value	10000
Lower specification Limit	9995
Upper Specification Limit	10005
Measured Value	10004
Measurement Error	4
Std. Uncert. (k=1)	0.08
Total Risk	0.00%
Upper Limit Risk	0.00%
Lower Limit Risk	0.00%
TUR =	**31.2296736**
Guard Band LSL	9995.164407
Guard Band USL	10004.83559

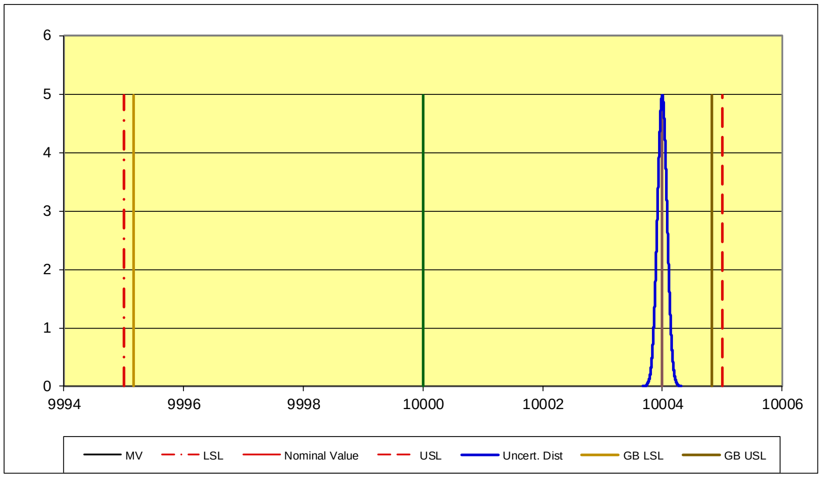

Figure 3: Graph Showing 10,004 as the measured value with a 31.23:1 TUR which is achieved by using a lab with low uncertainties

Calling an instrument "In Tolerance" is all about location, location, location. It's also about the uncertainty of the measurement, but a bad location is going to raise the Probability of False Accept (PFA) significantly. The probability of false accept is the likelihood of a lab calling a measurement "In Tolerance" when it is not. The location we are referring to is how close the measurement is to the nominal value. If the nominal value is 10,000 lbf and the instrument reads 10,004 lbf, the instrument bias is 4 lbf as shown in figure 4.

The larger the bias, the worse the location of the measurement. If we go back to our parking scenario, the worse the bias from nominal, the more likely one side of our automobile will be damaged, or maybe we are still "in tolerance", but must exit the vehicle from the other side. Higher TUR's help in producing guard banding limits where the calibration laboratory can still say the device being tested is within tolerance. A laboratory's scope of accreditation is a good indication of their capability to call an instrument in tolerance when any measurement bias is observed. Figure 5 shows the risk level increasing as we have switched calibration providers. The new provider has a higher CMC uncertainty component of 0.025 % than shown in figure 4 where the calibration provider had a 0.0016 % CMC uncertainty component. Everything else has remained the same.

Nominal Value	10000
Lower specification Limit	9995
Upper Specification Limit	10005
Measured Value	10004
Measurement Error	4
Std. Uncert. (k=1)	1.25
Total Risk	21.19%
Upper Limit Risk	21.19%
Lower Limit Risk	0.00%
TUR =	**1.999994666**
Guard Band LSL	9997.567193
Guard Band USL	10002.43281

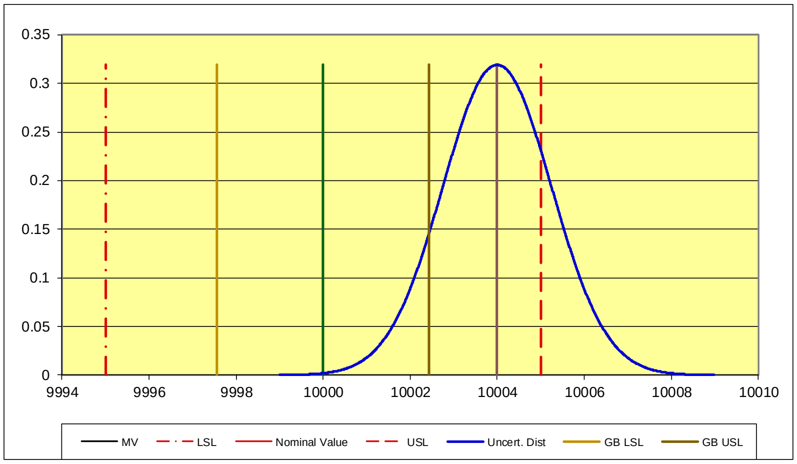

Figure 4: Graph Showing 10,004 as the measured value with a 1.99:1 TUR as the labs Expanded Uncertainty is higher than in figure 4

Why do we care about the location of the measurement if the device is within tolerance? If a device has a specification of 0.1 % of full scale and the calibrating laboratory reports a value within 0.1 % the device is "In Tolerance" right? The answer is and always will be. It depends on the uncertainty of the measurement and whether the lab performing the calibration followed the proper guidelines in determining the uncertainty of measurement (UOM) when making the statement of compliance.

For a refresher on why this matters, there is a certain standard called ISO/IEC 17025. The ISO/IEC 17025: 2005 standard in section 5.10.4.2 stated "When statements of compliance are made, the uncertainty of the measurement shall be taken into account." The ISO/IEC 17025: 2017 elaborates on this requirement in section 7.8.6.2 which states, "The laboratory shall report on the statement of conformity such that the statement clearly identifies –a) to which results the statement applies; and –b) which specifications, standard or parts thereof are met or not met; –c) the decision rule applied (unless it is inherent in the requested specification or standard)"

If you are following either version of the standard, this statement can be a big deal, and if the uncertainty of the measurement is significant, the lab performing the calibration is going to have to be very concerned with the location of the measurement. In fact, if their uncertainty of measurement is too high, they may not even be able to perform the calibration at all and if the measured value falls right on the specified tolerance line, the PFA will be 50 % or higher.

There are several techniques or methods to ensure a 2 % PFA requirement can be met. These techniques or methods are used to set acceptance limits to ensure the PFA is less than whatever is requested by the customer, which is usually 2 %. Setting acceptance limits that take the measurement uncertainty into account is often referred to as guard banding.

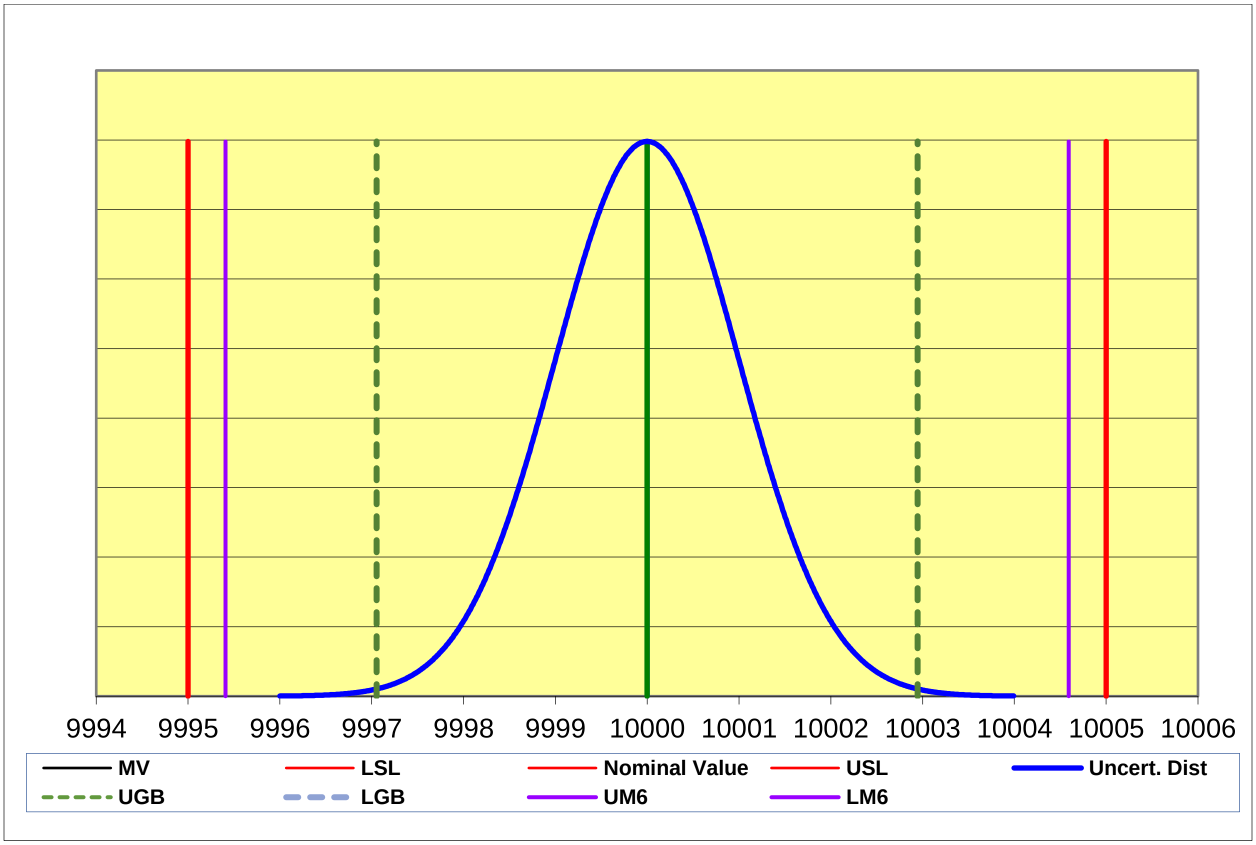

Figure 5: Graph Showing Specification Limits and Acceptance Limits for Both Method 5 and Method 6

Two Managed Risk Guard Banding Methods to Ensure the PFA is Less Than 2 %

ISO/IEC 17025:2017 section 7.8.6.2 states "The laboratory shall report on the statement of conformity such that the statement clearly identifies –a) to which results the statement applies; and –b) which specifications, standard or parts thereof are met or not met; –c) the decision rule applied (unless it is inherent in the requested specification or standard)".

In this paper, we are going to discuss three decision rules. Two of these rules, known as Method 5 and Method 6 are documented in ANSI/NCLI Z540.3 Handbook, and a third rule is something a lab may think about using to meet the criteria. The standard does not dictate what rules can or cannot be applied. It just requires that the calibration laboratory list the decision rule applied and that the laboratory discusses its decision criteria with the customer. There are several other guard banding and risk-based approaches than what is presented here. ILAC G8:09/2019 Guidelines on Decision

Rules and Statements of Conformity has several examples of decision rules and the corresponding risk associated with those rules.

Guard Band Method 5, Based on the Expanded Calibration Process Uncertainty. This method is simple as one subtracts the 95 % expanded process uncertainty from the tolerance limits. The above graphs in figures 1 through 4 use Method 5. It is very similar to the ILAC G8: 2009 rule.

ILAC-G8:03/2009 states if the specification limit is not breached by the measurement result plus the expanded uncertainty with a 95 % coverage probability, then compliance with the specification can be stated. The ILAC rule allows for a PFA of < 2.5 %.

Simply put, if one subtracts the expanded calibration process uncertainty from the upper limit of the specified tolerance, then the new acceptance limits will assure a PFA of less than 2.275 %. It is an interesting point as the ANSI/Z540.3 Handbook mentions 2 %, though the calculation gives a PFA of 2.275 %. One must assume some rounding took place. The only information needed to use Method 5 is the tolerance and the calibration process uncertainty formula in the figure below.

$$2 \text{ x } \mathbf{k}_{95\%} \left(\sqrt[2]{\left(\frac{\mathbf{CMC}}{\mathbf{k}_{\mathbf{CMC}}}\right)^2 + \left(\frac{\mathbf{Resolution}_{\mathbf{UUT}}}{\sqrt[2]{\mathbf{12}}}\right)^2 + \left(\frac{\mathbf{Repeatability}_{\mathbf{UUT}}}{\mathbf{1}}\right)^2 + \cdots (\mathbf{u}_{Other})^2} \right)$$

Figure 6: Calibration Process Uncertainty assuming 95 % confidence interval

Note: See ILAC-P14:09/2020 section 5.4 for requirements for calculating CMC and reporting measurement uncertainty. The requirements align very closely with the formula in Figure 7.

The downside of Method 5 is that the test limit is based on the worst-case PFA, which means they may be too aggressive, resulting in more false rejects from the reference laboratory. Being overly aggressive and needing to adjust more equipment lends one to look for an alternative method.

Guard Band Method 6, Based on Test Uncertainty Ratio:

This method is also simple as it depends only on the measurement uncertainty when compared with the specification limits of what is being calibrated. Per ANSI/NCSLI Z540.3 Handbook, "It makes use of an observation that for a given Test Uncertainty Ratio (TUR), there is a maximum PFA value for all values of the M&TE test point in-tolerance probability. Applying a guard band based on this maximum PFA value, and the corresponding TUR ensures that the PFA is 2 % or less regardless of the in-tolerance probability." It also results in guard bands with acceptance limits much larger than that of method 5.

The acceptance limit in Method 6 is given by:

$$A_{2\%} = L - U_{95\%} \times M_{2\%}$$

where:

$A_{2\%}$ = Acceptance limit used to achieve a maximum 2 % PFA

L = Tolerance limit (generally the manufacturer's specification)

$U_{95\%}$ = Calibration process 95 % expanded uncertainty

$M_{2\%}$ = The multiplier of the 95 % expanded calibration process uncertainty that provides a guard band to ensure 2 % PFA

NOTE: In other methods described in this appendix, the symbol A is used to describe the acceptance limits for any specific application. In this method, $A_{2\%}$ is being used to be consistent with the paper cited.

Combining the expressions for $M_{2\%}$ and $A_{2\%}$ results in:

$$A_{2\%} = L - U_{95\%} \times \left[1.04 - e^{(0.38\log(TUR) - 0.54)}\right]$$

The downside of Method 6 is it only works with TUR ratios of 0.76:1 through 4.5:1. Any ratio higher or lower will cause errors with not calculating the acceptance limits properly.

Comparing Method 5 versus Method 6

Below is a table using the same 10,000 lbf device, using the same variables as shown in figures 2-4, which are a 0.01 resolution, and a CMC uncertainty component of 0.0016 % from Morehouse who was used as the reference laboratory resulting in a 0.08 lbf calibration process at the 10,000 lbf pt.

DIFFERENCE IN ACCEPTANCE LIMITS METHOD 5 VS METHOD 6

Force or Torque Applied	Avg Instrument Reading	Method 6 AL	PASS/FAIL	Method 5 AL	PASS/FAIL	% Diff in AL
1000	1000.00	5.00	PASS	4.98	PASS	0.33%
2000	2000.00	5.00	PASS	4.97	PASS	0.63%
3000	3000.00	5.00	PASS	4.95	PASS	0.94%
4000	4000.00	5.00	PASS	4.93	PASS	1.25%
5000	5000.00	5.00	PASS	4.92	PASS	1.57%
6000	6000.00	4.99	PASS	4.90	PASS	1.88%
7000	7000.00	4.99	PASS	4.88	PASS	2.19%
8000	8000.00	4.99	PASS	4.87	PASS	2.50%
9000	9000.00	4.99	PASS	4.85	PASS	2.81%
10000	10000.00	4.99	PASS	4.84	PASS	3.13%

Figure 7: Difference in Acceptance Limits Method 5 versus Method 6 using a Reference Standard with an Expanded Uncertainty of 0.0016 %

DIFFERENCE IN ACCEPTANCE LIMITS METHOD 5 VS METHOD 6						
Force or Torque Applied	Avg Instrument Reading	**Method 6 AL**	PASS/FAIL	**Method 5 AL**	PASS/FAIL	**% Diff in AL**
1000	1000.00	**4.99**	PASS	**4.74**	PASS	**4.89%**
2000	2000.00	**4.97**	PASS	**4.49**	PASS	**9.80%**
3000	3000.00	**4.96**	PASS	**4.23**	PASS	**14.74%**
4000	4000.00	**4.95**	PASS	**3.97**	PASS	**19.70%**
5000	5000.00	**4.93**	PASS	**3.72**	PASS	**24.69%**
6000	6000.00	**4.84**	PASS	**3.46**	PASS	**28.45%**
7000	7000.00	**4.72**	PASS	**3.20**	PASS	**32.10%**
8000	8000.00	**4.59**	PASS	**2.95**	PASS	**35.84%**
9000	9000.00	**4.46**	PASS	**2.69**	PASS	**39.71%**
10000	10000.00	**4.32**	PASS	**2.43**	PASS	**43.74%**

Figure 8: Difference in Acceptance Limits Method 5 versus Method 6 using a Reference Standard with an Expanded Uncertainty of 0.025 %

DIFFERENCE IN ACCEPTANCE LIMITS METHOD 5 VS METHOD 6						
Force or Torque Applied	Avg Instrument Reading	**Method 6 AL**	PASS/FAIL	**Method 5 AL**	PASS/FAIL	**% Diff in AL**
1000	1000.00	**4.97**	PASS	**4.49**	PASS	**9.80%**
2000	2000.00	**4.95**	PASS	**3.97**	PASS	**19.70%**
3000	3000.00	**4.84**	PASS	**3.46**	PASS	**28.45%**
4000	4000.00	**4.59**	PASS	**2.95**	PASS	**35.84%**
5000	5000.00	**4.32**	PASS	**2.43**	PASS	**43.74%**
6000	6000.00	**4.04**	PASS	**1.92**	PASS	**52.48%**
7000	7000.00	**3.74**	PASS	**1.41**	PASS	**62.40%**
8000	8000.00	**3.43**	PASS	**0.89**	PASS	**73.97%**
9000	9000.00	**3.11**	PASS	**0.38**	PASS	**87.81%**
10000	10000.00	**2.78**	PASS	**-0.13**	FAIL	**104.83%**

Figure 9: Difference in Acceptance Limits Method 5 versus Method 6 using a Reference Standard with an Expanded Uncertainty of 0.05 %

When we analyze the data in Figures 8 through 10, it becomes apparent that the differences between Method 5 and Method 6 start to become quite drastic as the calibration process uncertainty increases. The CMC uncertainty component of the reference laboratory impacts the calibration process uncertainty, the resolution of the Test Instrument, and possibly the repeatability of the Test Instrument, which may or may not have been included in the calibration process uncertainty. The laboratory with the low CMC uncertainty component in Figure 8, shows the least amount of % difference from using Method 5, however, the formulas are based on the measurement process uncertainty, which includes the UUT's resolution and repeatability.

If the resolution and the repeatability of the UUT were to increase, the % difference would increase. Method 5 is the most affected as we subtract the measurement process uncertainty from the upper specification limits, and add it to the lower specification limit to obtain our acceptance limits. Figure 10 shows that the calibration laboratory would not make a conformity assessment using Method 5 at the last calibrated test point. However, using Method 6 allows that same laboratory to make a statement of conformity, assuming the measured value falls within the specified tolerance limits.

Conclusion

Any method used for calculating PFA will have both positive and negatives associated with implementation. The ISO/IEC 17025:2017 standard better addresses measurement risk by requiring the laboratory to report which specifications are not met and the decision rule applied. The decision rule applied needs to consider the location of the measurement for reporting False Accept Risk (PFA).

Throughout this paper, the author has demonstrated that TUR only shows the ratio of the specified tolerance compared to the calibration process uncertainty. If the ratio is too large, a laboratory may not be able to make a statement of conformance with complying with ISO/IEC 17025:2017. Furthermore, the author shows why a 4:1 or better TUR is not enough and stresses the importance of analyzing the measurement's location to ensure the measured value falls within the acceptance limits calculated by the accepted guard banding method used.

The best chance of continually meeting tolerance requirements is to use a reference lab (Calibration vendor) with the lowest CMC uncertainty component that replicates how the instrument is used. Also, the end-user must purchase the right equipment capable of continually achieving the desired result or adjust the tolerance appropriately.

Annex (Sample Calculation of TUR)

Example: A customer sends a **10,000 lbf** load cell for calibration with an accuracy specification of **± 0.05 % of full scale**. The calibration provider uses a Universal Calibrating Machine to perform the calibration. When **10,000 lbf** is applied, the unit reads **10,001 lbf**. The display resolution is **1 lbf.**

Step 1: Calculate the numerator

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 11: TUR Formula Nominator

The device is a **10,000 lbf** load cell with an accuracy specification of **± 0.05 %**

10,000 * 0.0005 = ± 5 lbf

The upper specification limit is 10,000 + 5 = 10,005 lbf

The lower specification limit is 10,000 – 5 = 9,995 lbf

Therefore, the Span of the ±Tolerance is 10,005 – 9,995 = 10 lbf

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 12: TUR Formula with the Numerator added

Step 2: Calculate the denominator

Everything is calculated to **1 standard deviation (Standard Uncertainty)** for this calculation.
Calibration and Measurement Capability (CMC)

$$TUR = \frac{\text{Span of the} \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 13: CMC portion of the denominator

CMC is the uncertainty at the calibrated force. The Universal Calibrating Machine has an uncertainty of **0.02 % at 10,000 lbf**.

The CMC is **10,000 * 0.0002 = 2 lbf**

k_{CMC} **is 2**, which was listed on the calibration provider's certificate.

Dividing the CMC by 2, the standard uncertainty is reported at **one standard deviation**. In most cases, the **CMC uncertainty component is reported at approximately 95 %**, and **a coverage factor of** $k = 2$ is used.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 14: TUR Formula with CMC added

UUT Resolution

$$\text{TUR} = \frac{\text{Span of the} \pm \text{Tolerance}}{2\,x\,k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 15: Resolution portion of the denominator

Resolution$_{UUT}$ for force instrument is calculated by dividing the force applied by the output at applied force and then multiplying this by the instrument's readability.

The **Resolution$_{UUT}$** is **(10,000 lbf / 10,000 lbf) * 1 = 1 lbf**

To convert **1 lbf** resolution to standard uncertainty, it is either divided by the **square root of 12**, or the square root of 3 depending on the Type of resolution.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 16: TUR Formula with Resolution added

Repeatability

$$TUR = \frac{\text{Span of the } \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 17: Repeatability portion of the denominator

For this example, **five replicate readings** are taken.

Repeatability is obtained by applying a force of **10,000 lbf** to the **Unit Under Test (UUT)** five times, and the sample standard deviation of five replicated measurements is calculated.

Repeatability of sample size five: **(10,000, 10,001, 10,000, 10,001, 10,001) = 0.54772**
Since the repeatability is already expressed as one standard deviation, the divisor is 1.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{0.54772}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 18: TUR Formula with Repeatability added

Other Error Sources

$$TUR = \frac{\text{Span of the } \pm \text{Tolerance}}{2 \times k_{95\%}\left(\sqrt[2]{\left(\frac{CMC}{k_{CMC}}\right)^2 + \left(\frac{Resolution_{UUT}}{\sqrt[2]{12}}\right)^2 + \left(\frac{Repeatability_{UUT}}{1}\right)^2 + \cdots (u_{Other})^2}\right)}$$

Figure 19: Other error sources in the denominator

Other error sources attributed to the **CPU** can be considered for the **UUT**. Some examples are environmental influences, error in correction factors, etc. For this example, other error sources are inherent in repeatability and **CMC**.

$$TUR = \frac{10\ lbf}{2\ x\ k_{95\%}\left(\sqrt[2]{\left(\frac{2\ lbf}{2}\right)^2 + \left(\frac{1\ lbf}{\sqrt[2]{12}}\right)^2 + \left(\frac{0.54772}{1}\right)^2}\right)}$$

Figure 20: TUR Formula with all error sources added

Calculate the Denominator

Sum of all the contributors = **SQRT((2/2)^2+(1/3.464)^2+(0.54772/1)^2) = 1.1762**

$$\textbf{TUR} = \frac{10\ lbf}{2\ x\ k_{95\%}\ (1.1762)}$$

Figure 21: TUR Calculated

The specification of **10 lbf** is divided by: **2 * k at 95 %** Calibration Process Uncertainty (**k = 2** for this example)

$$\textbf{TUR} = \frac{10\ lbf}{2\ x\ 2.35231} \qquad \textbf{TUR} = \frac{10\ lbf}{4.70462}$$

Figure22: TUR Calculated

TUR = 2.1256

Want to learn more?

Henry Zumbrun and Dilip Shah teach classes together at Morehouse Instrument Company about twice a year where the participants can learn more about the proper practices to ensure measurements are compliant to the ISO/IEC 17025:2017 and provide the tools to help minimize measurement errors.

References

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